LAGRANGE METHOD

microeconomics

(21) Assume a households utility function is $U(x,y) = x^{0,4} \cdot y^{0,6}$ and it's income is $I = 100$ and the price for good y is $P_y = 5$.
Derive formally the households demand function for good x.

Given information:

utility function $U(x,y) = x^{0.4} \cdot y^{0.6}$

Income $I = 100$

Price for good y $P_y = 5$

$U(x,y) = x^{\alpha} \cdot y^{1-\alpha}$

The Lagrangian is:

$$\mathcal{L} = \underbrace{x^{0,4} \cdot y^{0,6}}_{\text{utility function}} - \lambda \overbrace{(P_x \cdot x + 5y - 100)}^{\substack{\text{constrains} = 0 \\ (P_x \cdot x + P_y \cdot y - \text{Income})}}$$

The partial derivatives are:

Price for good x times quantity → not given

$$\frac{\partial \mathcal{L}}{\partial x} = 0{,}4 \cdot \frac{U}{x} - \lambda \cdot P_x \cdot x = 0$$

Information is given, see Langrangian

$$\frac{\partial \mathcal{L}}{\partial y} = 0{,}6 \cdot \frac{U}{y} - 5\lambda = 0$$

$$\boxed{\frac{\partial \mathcal{L}}{\partial \lambda} = P_x \cdot x + 5\lambda - 100 = 0}$$

→ All information put together

Then:

$$x = \frac{0{,}4 \cdot 100 \text{ Euros}}{P_x \cdot x \text{ Euros/units}} = \boxed{\frac{40}{P_x \cdot x}} = \text{Demand function for good } x$$

and:

$$y = \frac{0{,}6 \cdot 100 \text{ Euros}}{5 \text{ Euros/units}} = 12$$